Published by Institute of Physics Publishing for SISSA
Received: *July 26, 2007*
Accepted: *August 21, 2007*
Published: *September 3, 2007*

Kappa-symmetry and coincident D-branes

Paul Howe,[a] **Ulf Lindström**[bcd] **and Linus Wulff**[e]

[a] *Department of Mathematics, King's College, Strand, London, WC2R 2LS, U.K.*
[b] *Department of Theoretical Physics, Box 803, SE-751 08, Uppsala, Sweden*
[c] *HIP-Helsinki Institute of Physics, University of Helsinki, P.O. Box 64 FIN-00014, Suomi-Finland*
[d] *NORDITA, Roslagstullsgatan 23, SE-106 91 Stockholm, Sweden*
[e] *Dipartimento di Fisica "Galileo Galilei", Università degli Studi di Padova & INFN, Sezione di Padua, via F. Marzolo 8, 35131 Padova, Italia*
E-mail: paul.howe@kcl.ac.uk, Ulf.Lindstrom@teorfys.uu.se, linus@physto.se

Abstract: A kappa-symmetric action for coincident D-branes is presented. It is valid in the approximation that the additional fermionic variables, used to incorporate the non-abelian degrees of freedom, are treated classically. The action is written as a Bernstein-Leites integral on the supermanifold obtained from the bosonic worldvolume by adjoining the extra fermions. The integrand is a very simple extension of the usual Green-Schwarz action for a single brane; all symmetries, except for kappa, are manifest, and the proof of kappa-symmetry is very similar to the abelian case.

Keywords: D-branes, Supersymmetric Effective Theories.

Contents

1. Introduction

The dynamics of a set of coincident D-branes is an intriguing problem in string theory. It involves a non-abelian version of Dirac-Born-Infeld theory as well as non-commutative geometrical ideas. There have been many papers written on the topic from various points of view, although a completely satisfactory theory has not emerged as yet. In this paper, we follow up an approach developed in two previous papers [1, 2] in which we made use of the idea that the Chan-Paton factors for open strings can be described mathematically by boundary fermions living at the ends of the string [3−6]. In our first paper we looked at what happens when one demands kappa-symmetry for an open superstring with boundary fermions and found that it implies that the dynamics of the brane on which the string ends is described by a generalised superembedding, where the super worldvolume of the brane is extended by a set of additional odd coordinates corresponding to the boundary fermions. There is an abelian gauge field on this space which gives rise to a non-abelian one when expanded out in the additional fermi coordinates. The requirement of kappa-symmetry leads to constraints on the superembedding and the gauge field strength which generalise those for a single brane [7, 8] and which imply the equations of motion for the brane system. In the second paper, we presented an action for a bosonic brane with additional fermi variables and showed that it is invariant under diffeomorphisms of the extended worldvolume and under gauge transformations of the target space RR potentials. We also showed how one could derive the Myers action [9−11] by first going to the physical gauge, quantising the fermions naively, thereby converting functions of fermions into matrices, and by replacing the fermi integral with the symmetrised trace. The current paper can

JHEP09(2007)010

be thought of as a synthesis of the previous two in that here we discuss an action for supersymmetric coincident branes, again in the approximation of classical additional fermi variables. We write this action as a Bernstein-Leites integral [12] over the extension of the bosonic worldvolume, $\widehat{M}_0$. This formalism seems to be perfectly suited to this problem and allows us to write down an action which is manifestly invariant under diffeomorphisms of $\widehat{M}_0$ and under symmetries of the target space, unlike our previous action for bosonic branes. It is also straightforward to prove that it is kappa-symmetric. Indeed, the action is a very natural generalisation of the usual Green-Schwarz action for a single D-brane and gives a very nice *a posteriori* justification for the Myers action. A preliminary version of the proof of kappa-symmetry given here, based on our old formalism, was given in [13].

The paper is organised as follows: in section 2 we review some results from the superembedding formalism which we shall need for our proof of kappa-symmetry; in section 3 we present the Dirac-Born-Infeld and Wess-Zumino parts of the action as Bernstein-Leites integrals and in section 4 we prove that the sum of these two terms is kappa-symmetric. We summarise our results in section 5 and discuss how our formalism might be developed further and how it relates to various other approaches in the literature.

2. The geometrical framework

As discussed in [1] the geometry of coincident superbranes, in the approximation of treating the boundary fermions classically, is described by a generalised superembedding $\widehat{f}:\widehat{M}\to\underline{M}$ from the extended superworldvolume $\widehat{M}$ to the target superspace which we shall take to be that of on-shell IIB supergravity in this paper. This is a generalisation of the usual superembedding formalism for single branes. The Green-Schwarz action for the dynamics of this system will be given as an integral over $\widehat{M}_0$, where M_0 (coordinates x^m) is the body of the super worldvolume M (coordinates $z^M=(x^m,\theta^\mu)$). The spaces $\widehat{M}_0$ (coordinates $x^{\widehat{m}}=(x^m,\xi^{\dot\mu})$) and $\widehat{M}$ (coordinates $z^{\widehat{M}}=(z^M,\xi^{\dot\mu})$) are obtained from M_0 and M by adjoining a set of q additional fermionic variables $\xi^{\dot\mu}$ which arise from boundary fermions on the string.[1] The various worldvolume spaces are related as follows:

$$\begin{array}{ccc} M & \to & \widehat{M} \\ \uparrow & & \uparrow \\ M_0 & \to & \widehat{M}_0 \end{array} \tag{2.1}$$

where horizontal arrows indicate extension with additional fermionic variables $\xi^{\dot\mu}$ representing the boundary fermions, and vertical arrows indicate addition of supersymmetry, i.e., adding θ^μ. The corresponding diagram for the coordinates is

$$\begin{array}{ccc} (z^M)=(x^m,\theta^\mu) & \to & (z^M,\xi^{\dot\mu})=(x^m,\theta^\mu,\xi^{\dot\mu}) \\ \uparrow & & \uparrow \\ (x^m) & \to & (z^M)=(x^m,\xi^{\dot\mu}) \end{array} \tag{2.2}$$

[1]There is a slight change of notation compared to our previous papers; the boundary fermions are denoted $\xi^{\dot\mu}$ instead of $\eta^{\widehat{\mu}}$. Hatted indices indicate standard ones extended by these fermions.

All of the above spaces, as well as the target superspace, are equipped with preferred bases in the tangent spaces which will be denoted by letters from the beginning of the alphabet; thus the preferred basis forms on $\widehat{M}$ are $E^{\widehat{A}} = (E^a, E^\alpha, E^{\dot\alpha})$, while those of $\underline{M}$ are denoted $E^{\underline{A}} = (E^{\underline{a}}, E^{\underline{\alpha}})$. To avoid confusion we shall use small letters for the bases of M_0 and $\widehat{M}_0$; thus the preferred basis forms of the latter space are denoted $e^{\widehat{a}} = (e^a, e^{\dot\alpha})$.

The geometry of the tangent bundle of $\widehat{M}$ is chosen such that it splits invariantly into three corresponding to the three types of indices. Thus the structure group has the usual superspace type (spin group times internal symmetry group) in the (E^a, E^α) sector while it is taken to be $\mathrm{SO}(q)$ in the $E^{\dot\alpha}$ sector, where q is the number of boundary fermions. We introduce connections (Ω) and covariant derivatives (∇) and define the torsion (T) and curvature forms (R) in the usual way. In addition there is an abelian gauge field A with corresponding field strength K defined by

$$K := dA - \widehat{f}^* B \tag{2.3}$$

where B is the Neveu-Schwarz two-form potential on $\underline{M}$. The geometry of $\widehat{M}$ is determined by the superembedding. The derivative of $\widehat{f}$ is the superembedding matrix $E_{\widehat{A}}{}^{\underline{A}}$ defined by

$$E_{\widehat{A}}{}^{\underline{A}} := E_{\widehat{A}}{}^{\widehat{M}} \partial_{\widehat{M}} z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}. \tag{2.4}$$

where $E_M{}^A$, $E_A{}^M$ denotes the supervielbein and its inverse. We shall use two real fermions of the same chirality to describe the odd coordinates of $\underline{M}$; accordingly, the preferred basis forms are written $E^{\underline{\alpha}} = (E^{\alpha 1}, E^{\alpha 2})$. We now impose the following constraints on the superembedding matrix:

$$\begin{aligned} E_\alpha{}^{\underline{b}} &= 0 & E_a{}^{\underline{b}} &= u_a{}^{\underline{b}} \\ E_\alpha{}^{\beta 1} &= u_\alpha{}^\beta & E_\alpha{}^{\beta 2} &= h_\alpha{}^{\gamma'} u_{\gamma'}{}^\beta \\ E_a{}^{\beta 1} &= 0 & E_a{}^{\beta 2} &= h_a{}^{\gamma'} u_{\gamma'}{}^\beta \end{aligned} \tag{2.5}$$

where $u_\alpha{}^\beta$ is an element of $\mathrm{Spin}(1,9)$ with corresponding Lorentz group element $(u_a{}^{\underline{a}}, u_{a'}{}^{\underline{a}})$. In fact, the primed indices denote indices normal to M in $\underline{M}$, but note that there are no primed dotted indices. The primed spinor indices are no different to the unprimed ones as far as representations of the spin group are concerned and there is no need to distinguish them. The above constraints are the direct analogies of the abelian ones; the main one is the first, $E_\alpha{}^{\underline{a}} = 0$, since the others correspond to choices. The field $h_\alpha{}^{\beta'}$ is related to the field strength of the gauge field, while $h_a{}^{\gamma'}$ is essentially the bosonic derivative on the brane of the transverse fermions. In addition we choose

$$E_{\dot\alpha}{}^{\underline{b}} = h_{\dot\alpha}{}^{c'} u_{c'}{}^{\underline{b}}; \qquad E_{\dot\alpha}{}^{\beta 1} = 0; \qquad E_{\dot\alpha}{}^{\beta 2} = h_{\dot\alpha}{}^{\gamma'} u_{\gamma'}{}^\beta \ . \tag{2.6}$$

The fields $h_{\dot\alpha}{}^{a'}$ and $h_{\dot\alpha}{}^{\alpha'}$ can be thought of as the derivatives of the transverse bosons and fermions respectively with respect to the boundary fermion variables. There are also constraints on the gauge field strength tensor K. These are:

$$\begin{aligned} K_{AB} &= \begin{cases} K_{ab} := \mathcal{F}_{ab} \\ K_{\alpha B} = 0 \end{cases} \\ K_{\dot\alpha B} &= 0 \\ K_{\dot\alpha\dot\beta} &= \delta_{\dot\alpha\dot\beta} \end{aligned} \tag{2.7}$$

The first of equations (2.7) is a direct generalisation of the abelian gauge field constraint for a single brane [14] while the others have the effect of excluding unphysical degrees of freedom. The requirement that $K_{\dot\alpha\dot\beta}$ be non-singular is necessary in order that the abelian field strength should be equivalent to a non-abelian gauge-field (on M) when expanded in powers of ξ. Equation (2.7) can be written more succinctly as

$$K = \mathcal{I} + \mathcal{F}\,, \tag{2.8}$$

where

$$\mathcal{F} := \frac{1}{2}E^b E^a \mathcal{F}_{ab}\,, \tag{2.9}$$

and where $\mathcal{I}$ is the unit two-form in the dotted sector,

$$\mathcal{I} := \frac{1}{2}E^{\dot\beta}E^{\dot\alpha}\delta_{\dot\alpha\dot\beta}\;. \tag{2.10}$$

The details of the induced geometry on $\widehat{M}$ are determined from the torsion equation,

$$2\nabla_{[\widehat{A}}E_{\widehat{B}]}{}^{\underline{C}} + T_{\widehat{A}\widehat{B}}{}^{\widehat{C}}E_{\widehat{C}}{}^{\underline{C}} = (-1)^{(\widehat{B}+\underline{B})\widehat{A}}E_{\widehat{B}}{}^{\underline{B}}E_{\widehat{A}}{}^{\underline{A}}T_{\underline{AB}}{}^{\underline{C}}\,, \tag{2.11}$$

and from the Bianchi identity for K,

$$\begin{aligned} 3\nabla_{[\widehat{A}}K_{\widehat{B}\widehat{C}]} + 3T_{[\widehat{A}\widehat{B}}{}^{\widehat{D}}K_{|\widehat{D}|\widehat{C}]} &= -H_{\widehat{A}\widehat{B}\widehat{C}} \\ &:= -(-1)^{(\widehat{B}+\underline{B})\widehat{A}}(-1)^{(\widehat{C}+\underline{C})(\widehat{A}+\widehat{B})}E_{\widehat{C}}{}^{\underline{C}}E_{\widehat{B}}{}^{\underline{B}}E_{\widehat{A}}{}^{\underline{A}}H_{\underline{ABC}}\;. \end{aligned} \tag{2.12}$$

In (2.12) the vertical bars indicate that the enclosed index is excluded from the graded antisymmetrisation.

In order to solve these equations we need to specify the constraints on the IIB target space geometry [15]. In the string frame we may take

$$\begin{aligned} T^{\underline{a}} &= -\frac{i}{2}E^{\beta j}E^{\alpha i}\delta_{ij}(\gamma^{\underline{a}})_{\alpha\beta} \\ H &= -\frac{i}{2}E^{\underline{c}}E^{\beta j}E^{\alpha i}(\sigma^3)_{ij}(\gamma_{\underline{c}})_{\alpha\beta} + \frac{1}{3!}E^{\underline{c}}E^{\underline{b}}E^{\underline{a}}H_{\underline{abc}}\;. \end{aligned} \tag{2.13}$$

Here $i = 1,2$ is a Spin(2) index and σ^3 is the third Pauli matrix. There are other constraints which we shall not need, although, as shown in [16], the equations of motion of IIB supergravity follow from the standard constraint on the dimension zero torsion.

In order to discuss the Wess-Zumino term in the action we shall also need the RR field strengths, $G^{(2n+1)}$, $n = 1,\dots 5$, which are given by [17, 18]

$$\begin{aligned} G^{(2n+1)} &= ie^{-\phi}E^{\beta 2}E^{\alpha 1}(\gamma^{(2n-1)})_{\alpha\beta} - e^{-\phi}\Big(E^{\alpha 1}(\gamma^{(2n)}\nabla_2\phi)_\alpha - (-1)^n E^{\alpha 2}(\gamma^{(2n)}\nabla_1\phi)_\alpha\Big) \\ &\quad + \frac{1}{(2n+1)!}E^{\underline{a}_{2n+1}}\dots E^{\underline{a}_1}G_{\underline{a}_1\dots\underline{a}_{2n+1}}\,, \end{aligned} \tag{2.14}$$

where

$$\gamma^{(r)} := \frac{1}{r!}E^{\underline{a}_r}\dots E^{\underline{a}_1}\gamma_{\underline{a}_1\dots\underline{a}_r}\;. \tag{2.15}$$

2.1 The field $h_\alpha{}^{\beta'}$

Using the constraints $K_{\dot{\alpha}B} = K_{\alpha\widehat{B}} = 0$ in the $(\alpha\beta\widehat{C})$ component of the K Bianchi identity (2.12) we find

$$T_{\alpha\beta}{}^{\widehat{D}} K_{\widehat{D}\widehat{C}} = -H_{\alpha\beta\widehat{C}}\,. \tag{2.16}$$

Using the form of the generalised superembedding matrix (2.5) in the $(\alpha\beta)^{\underline{c}}$-component of the torsion equation, (2.11), we have

$$T_{\alpha\beta}{}^{c} E_c{}^{\underline{c}} + T_{\alpha\beta}{}^{\dot{\gamma}} E_{\dot{\gamma}}{}^{\underline{c}} = -i(\gamma^{\underline{c}} + h\gamma^{\underline{c}} h^{\mathrm{T}})_{\alpha\beta}\,. \tag{2.17}$$

The projections along the wordvolume and normal directions respectively give

$$T_{\alpha\beta}{}^{a} = -i(\gamma^a + h\gamma^a h^{\mathrm{T}})_{\alpha\beta} \tag{2.18}$$

and

$$T_{\alpha\beta}{}^{\dot{\gamma}} h_{\dot{\gamma}}{}^{a'} = -i(\gamma^{a'} + h\gamma^{a'} h^{\mathrm{T}})_{\alpha\beta}\,. \tag{2.19}$$

These two equations, together with (2.16), give

$$i(\gamma^d + h\gamma^d h^{\mathrm{T}})_{\alpha\beta}\mathcal{F}_{dc} = H_{\alpha\beta c} \tag{2.20}$$

and

$$H_{\alpha\beta\dot{\gamma}}\delta^{\dot{\gamma}\dot{\delta}} h_{\dot{\delta}}{}^{a'} = i(\gamma^{a'} + h\gamma^{a'} h^{\mathrm{T}})_{\alpha\beta}\,. \tag{2.21}$$

The $(\alpha\beta\widehat{C})$ component of the pull-back of H, from (2.13), is

$$H_{\alpha\beta\widehat{C}} = -iE_{\widehat{C}}{}^{\underline{c}}(\gamma_{\underline{c}} - h\gamma_{\underline{c}} h^{\mathrm{T}})_{\alpha\beta}\,, \tag{2.22}$$

so that the equations for h become

$$\begin{aligned}
\mathcal{F}_{ab}(\gamma^b + h\gamma^b h^{\mathrm{T}})_{\alpha\beta} &= (\gamma_a - h\gamma_a h^{\mathrm{T}})_{\alpha\beta} \\
h_{\dot{\gamma}}{}^{a'}\delta^{\dot{\gamma}\dot{\delta}} h_{\dot{\delta}}{}^{b'}(\gamma_{b'} - h\gamma_{b'} h^{\mathrm{T}})_{\alpha\beta} &= (\gamma^{a'} + h\gamma^{a'} h^{\mathrm{T}})_{\alpha\beta}\,.
\end{aligned} \tag{2.23}$$

Defining the antisymmetric matrix

$$M^{a'b'} := \delta^{\dot{\alpha}\dot{\beta}} h_{\dot{\alpha}}{}^{a'} h_{\dot{\beta}}{}^{b'} \tag{2.24}$$

and rearranging we get

$$\begin{aligned}
h\gamma^a h^{\mathrm{T}} &= \gamma^c((1-\mathcal{F})^{-1})_c{}^b (1+\mathcal{F})_b{}^a \\
h\gamma^{a'} h^{\mathrm{T}} &= -\gamma^{c'}((1-M)^{-1})_c{}^b (1+M)_b{}^a\,.
\end{aligned} \tag{2.25}$$

The solution to these equations can be written as

$$h = h_{\|} h_{\perp} \gamma_{(p+1)}\,, \tag{2.26}$$

where

$$\gamma_{(p+1)} \equiv \frac{1}{(p+1)!}\varepsilon_{a_0\cdots a_p}\gamma^{a_0\cdots a_p} \tag{2.27}$$

and where $h_\parallel$ and $h_\perp$ are spin transformations corresponding to the Lorentz and orthogonal transformations

$$\begin{aligned} L_a{}^b &= ((1-\mathcal{F})^{-1}(1+\mathcal{F}))_a{}^b \quad \in \mathrm{SO}(1,p) \\ L_{a'}{}^{b'} &= ((1-M)^{-1}(1+M))_{a'}{}^{b'} \in \mathrm{SO}(9-p)\,, \end{aligned} \tag{2.28}$$

which are written in the so-called Cayley parametrisation. They are given by [19]

$$\begin{aligned} h_\parallel &= \frac{1}{\sqrt{-\det\,(\eta+\mathcal{F})}}\text{Æ}\left(\frac{1}{2}\mathcal{F}_{ab}\gamma^{ab}\right) \\ h_\perp &= \frac{1}{\sqrt{\det\,(1+M)}}\text{Æ}\left(\frac{1}{2}M^{a'b'}\gamma_{a'b'}\right)\,, \end{aligned} \tag{2.29}$$

where the "antisymmetrised exponential" is defined by

$$\text{Æ}(X_{ab}\gamma^{ab}) := \sum_{n=0}\frac{1}{n!}X_{a_1b_1}\cdots X_{a_nb_n}\gamma^{a_1b_1\cdots a_nb_n}\,. \tag{2.30}$$

It is not hard to show that h can also be expressed as

$$h = \frac{1}{\sqrt{-\mathrm{sdet}\,(\eta+K)}}\sum_{n=0}\frac{1}{2^n n!}K_{\widehat{a}_1\widehat{b}_1}\cdots K_{\widehat{a}_n\widehat{b}_n}\gamma^{\widehat{a}_1\widehat{b}_1\cdots\widehat{a}_n\widehat{b}_n}\gamma_{(p+1)}\,, \tag{2.31}$$

where $\gamma_{\widehat{a}} := E_{\widehat{a}}{}^{\underline{a}}\gamma_{\underline{a}}$ and where the superdeterminant is taken over the subspace spanned by $(E^a, E^{\dot\alpha})$. Note that $\dot\alpha$ indices are raised and lowered by means of $\delta_{\dot\alpha\dot\beta}$ and not by $\eta_{\dot\alpha\dot\beta}$ where η is the metric induced from the bosonic target space metric,

$$\eta_{\widehat{a}\widehat{b}} := E_{\widehat{a}}{}^{\underline{a}}E_{\widehat{b}}{}^{\underline{b}}\eta_{\underline{ab}}\,. \tag{2.32}$$

Since $E_\alpha{}^{\underline{a}} = 0$ this metric is the non-vanishing part of the pull-back of $\eta_{\underline{ab}}$ onto the whole of the tangent space of $\widehat{M}$.

For future use we note that the gamma-matrix structure of h and $(h^T)^{-1}$ is

$$\begin{aligned} h &\sim \sum \gamma^{2m}\gamma'^{2l}\gamma_{(p+1)} \\ (h^T)^{-1} &\sim -\sum \widetilde{\gamma}^{2m}\widetilde{\gamma}'^{2l}\gamma_{(p+1)}\,, \end{aligned} \tag{2.33}$$

where γ' denotes matrices with primed indices and where, in the second line, the tilde denotes the index structure is $(\gamma)^\alpha{}_\beta$. In general we shall not distinguish the two types of gamma matrix except where it is useful for clarity.

2.2 Some useful torsion components

We record here some components of the torsion tensor which will be used in the proof of kappa-symmetry. For completeness we reproduce (2.18):

$$T_{\alpha\beta}{}^a = -i(\gamma^a + h\gamma^a h^{\mathrm{T}})_{\alpha\beta}\,. \tag{2.34}$$

From the $(\alpha\beta\dot{\gamma})$ component of the K Bianchi identity we find

$$\begin{aligned} T_{\alpha\beta\dot{\gamma}} &= -H_{\alpha\beta\dot{\gamma}} \\ \Rightarrow\ T_{\alpha\beta\dot{\gamma}} &= iE_{\dot{\gamma}}{}^{\underline{c}}(\gamma_{\underline{c}} - h\gamma_{\underline{c}}h^T)_{\alpha\beta} \\ &= i(\gamma_{\dot{\gamma}} - h\gamma_{\dot{\gamma}}h^T)_{\alpha\beta}\ . \end{aligned} \tag{2.35}$$

Using the $(\alpha b)^{\underline{c}}$ component of the torsion equation projected along the worldvolume we find

$$T_{\alpha b}{}^{c} = ih_{\alpha}{}^{\gamma}(\gamma^c)_{\gamma\beta}h_b{}^{\beta} := i(h\gamma^c h_b)_{\alpha}\ . \tag{2.36}$$

The other relevant dimension one-half torsion can be found from the $(\alpha\dot{\beta}\dot{\gamma})$ component of the K Bianchi identity (2.12); it is

$$T_{\alpha\dot{\beta}\dot{\gamma}} = -iE_{(\dot{\beta}}{}^{\underline{c}}h_{\alpha}{}^{\gamma}(\gamma_{\underline{c}})_{\gamma\beta}h_{\dot{\gamma})}{}^{\beta} := -i(h\gamma_{(\dot{\beta}}h_{\dot{\gamma})})_{\alpha}\,, \tag{2.37}$$

where we have used a choice of connection to set $T_{\alpha[\dot{\beta}\dot{\gamma}]} = 0$. We shall also need the fermionic derivatives of $\mathcal{F}_{bc}$ and $\eta_{\dot{\beta}\dot{\gamma}}$. The former can be found from the (αbc) component of the K Bianchi identity together with (2.36),

$$\nabla_{\alpha}\mathcal{F}_{bc} = 2i(h\gamma^d h_{[b})_{\alpha}(\eta_{c]d} + \mathcal{F}_{c]d})\,, \tag{2.38}$$

while the latter can be computed using the definition of $\eta_{\dot{\beta}\dot{\gamma}}$ and the $(\alpha\dot{\beta})^{\underline{c}}$ component of the torsion equation, along with (2.37) which allow one to find $\nabla_{\alpha}E_{\dot{\beta}}{}^{\underline{c}}$. A short calculation yields

$$\nabla_{\alpha}\eta_{\dot{\beta}\dot{\gamma}} = i(1+\eta)_{[\dot{\beta}}{}^{\dot{\delta}}(h\gamma_{|\dot{\delta}|}h_{\dot{\gamma}]})_{\alpha} - i(1-\eta)_{[\dot{\beta}}{}^{\dot{\delta}}(h\gamma_{\dot{\gamma}]}h_{\dot{\delta}})_{\alpha}\ . \tag{2.39}$$

3. The action

In [2] we presented the Dirac-Born-Infeld and Wess-Zumino terms in the action for a set of coincident bosonic branes in terms of standard superspace integrals over the supermanifold $\widehat{M}_0$. However, it turns out that the superspace integration formalism of Bernstein and Leites is much more suitable for this task [12]. Bernstein-Leites integration has been used previously in a string theory context; see, for example, [20, 21]. The idea is that, instead of integrating over $\widehat{M}_0$, one should integrate over $\Pi T\widehat{M}_0$ where Π denotes Grassmann parity flip in the fibres of the tangent bundle $T\widehat{M}_0$. That is, one integrates over (x,ξ) and $(dx, d\xi)$ where $d\xi$ (dx) are regarded as even (odd) variables. The integrands are pseudo-differential forms, that is, inhomogeneous forms which can involve arbitrary functions of the even variables. The integral over dx is given by the standard Berezin rules and therefore projects out the top form in dx, while the integral over $d\xi$ is a formal version of a standard integral. In the D-brane case it turns out that this part of the integration is Gaussian and easily computed. As we shall see it gives rise to the contraction of forms with the matrix commutator of the non-abelian transverse coordinates which appears in the Myers WZ term.

The basic integration formula we shall need is the following: let y^r be a set of q real commuting variables and A a real, symmetric, invertible $q \times q$ matrix, and let $P(y)$ be a polynomial in y, then

$$\int dy\; e^{-\frac{1}{2}y^T A^{-1} y} P(y) = e^{\frac{1}{2}i_A} P(y)|_{y=0}\,, \tag{3.1}$$

where i_A denotes the differential operator $A^{rs}\partial_r\partial_s$ and where we have absorbed the square root of the determinant of A and factors of π into the normalisation of the integral. In particular, if A is the unit matrix and P is homogeneous of degree $2n$,

$$P = \frac{1}{(2n)!} P_{r_1\ldots r_{2n}} y^{r_1} \ldots y^{r_{2n}}\,, \tag{3.2}$$

this formula picks out the multi-trace of P, given by

$$\begin{aligned}\int dy\; e^{-\frac{1}{2}y^T y} P(y) &= \frac{1}{2^n n!}\delta^{r_1 r_2}\ldots\delta^{r_{2n-1}r_{2n}} P_{r_1\ldots r_{2n}} \\ &:= \frac{1}{2^n n!}\delta^{r_1\ldots r_{2n}} P_{r_1\ldots r_{2n}}\end{aligned} \tag{3.3}$$

The action will take the form of a Bernstein-Leites integral on $\widehat{M}_0$ of a pseudo-differential form on the same space. However, in the supersymmetric context it is more convenient to think of the action as a pseudo-form on $\widehat{M}$, bearing in mind that it is to be pulled back to $\widehat{M}_0$ before evaluation of the integral. If we regard $\widehat{M}_0$ as being embedded in $\widehat{M}$ then E^a and $E^{\dot{\alpha}}$ will pull back to e^a and $e^{\dot{\alpha}}$ respectively while E^α pulls back to both of them,

$$E^\alpha \to e^a e_a{}^\alpha + e^{\dot{\alpha}} e_{\dot{\alpha}}{}^\alpha \text{ on } \widehat{M}_0\,. \tag{3.4}$$

3.1 The DBI term

The Dirac-Born-Infeld pseudo-form is

$$L_{\rm DBI} = e^{-\mathcal{I}} e^{-\phi} \varepsilon_{(p+1)} L_0 \tag{3.5}$$

where $\varepsilon_{(p+1)}$ is the bosonic volume form,

$$\varepsilon_{(p+1)} = \frac{1}{(p+1)!} E^{a_{p+1}} \ldots E^{a_1} \varepsilon_{a_1\ldots a_{p+1}}\,, \tag{3.6}$$

and where L_0 is the Dirac-Born-Infeld function,

$$L_0 := \sqrt{-\mathrm{sdet}(\eta + K)}\,. \tag{3.7}$$

The superdeterminant here is understood to be over the subspace spanned by $E^{\widehat{a}}$,

$$\mathrm{sdet}(\eta + K) = \det(\eta_{ab} + \mathcal{F}_{ab})\det{}^{-1}(\delta_{\dot{\alpha}\dot{\beta}} + \eta_{\dot{\alpha}\dot{\beta}})\,. \tag{3.8}$$

It is trivial to carry out the integration over dx and $d\xi$; we find

$$\begin{aligned}\int Dx\, D\xi\, D(dx)\, D(d\xi)\; L_{\rm DBI} &= \int Dx\, D\xi\, D(e^a)\, D(e^{\dot{\alpha}})\; \mathrm{sdet}\, e\, L_{\rm DBI} \\ &= \int dx\, d\xi\; \mathrm{sdet}\, e\, e^{-\phi}\sqrt{-\mathrm{sdet}\,(\eta + K)}\,,\end{aligned} \tag{3.9}$$

where the final expression is a standard integral over $\widehat{M}_0$. It agrees with the one given in [2], except for the dilaton factor which was omitted there.

3.2 The Wess-Zumino term

The Wess-Zumino pseudo-form is given by

$$L_{\rm WZ} := e^{-K}C \tag{3.10}$$

where C is the sum of the RR potentials pulled back to $\widehat{M}$. Notice that we do not have to project out a particular form component as the integral takes care of this.

When we pull-back $L_{\rm WZ}$ to $\widehat{M}_0$ it will give rise to a pseudo-form of the type $e^{-\mathcal{I}}\omega$ where ω has $(p{+}1)$ even indices and $2n$ odd indices (since any other terms would integrate to zero). We therefore have to evaluate integrals of the form $\int Dx\, D\xi\, D(e^a)\, D(e^{\dot\alpha})\, \mathrm{sdet}\, e\, e^{-\mathcal{I}}\omega_{p+1,2n}$. The integrations over e^a and $e^{\dot\alpha}$ are easily done and we get

$$\int Dx\, D\xi\, D(e^a)\, D(e^{\dot\alpha})\, \mathrm{sdet}\, e\, e^{-\mathcal{I}}\omega_{p+1,2n} = -\int dx\, d\xi\, \mathrm{sdet}\, e\, \varepsilon^{a_1\ldots a_{p+1}} \left(e^{\frac{1}{2}i_\delta}\omega\right)_{a_1\ldots a_{p+1}}\ , \tag{3.11}$$

where

$$(i_\delta \omega_{0,2n})_{\dot\alpha_3\ldots\dot\alpha_{2n}} := \delta^{\dot\alpha_1\dot\alpha_2}\omega_{\dot\alpha_1\ldots\dot\alpha_{2n}}\ . \tag{3.12}$$

The Wess-Zumino part of the action is therefore given by

$$\int Dx\, D\xi\, D(e^a)\, D(e^{\dot\alpha})\, \mathrm{sdet}\, e\, L_{\rm WZ} = -\int dx\, d\xi\, \mathrm{sdet}\, e\, \varepsilon^{a_1\ldots a_{p+1}} \left(e^{\frac{1}{2}i_\delta}e^{-\mathcal{F}}C\right)_{a_1\ldots a_{p+1}}\ . \tag{3.13}$$

This is our final expression; it is similar to that given in [2] except that it is written in a frame basis rather than a coordinate one. As such it is manifestly covariant with respect to diffeomorphisms of $\widehat{M}_0$ whereas it took some work to show that the coordinate version has this property. A proof of the equivalence of the two is given in [13]. It is easy to see how the terms involving higher rank forms appear, however. For example, consider a $(p+1,2)$-form ω of the type appearing in the integrand of (3.13). If we consider ω as a form on $\widehat{M}$ pulled back from $\underline{M}$ we have

$$\begin{aligned} i_\delta\omega &= \frac{1}{(p+1)!}E^{a_{p+1}}\ldots E^{a_1}\delta^{\dot\alpha\dot\beta}\omega_{a_1\ldots a_{p+1}\dot\alpha\dot\beta} \\ &= \frac{1}{(p+1)!}E^{a_{p+1}}\ldots E^{a_1}M^{b'c'}u_{b'}{}^{\underline{b}}u_{c'}{}^{\underline{c}}\omega_{a_1\ldots a_{p+1}\underline{bc}}\ . \end{aligned} \tag{3.14}$$

We can think of $M^{a'b'}$ as being essentially the Poisson bracket of the transverse coordinates which will become the commutator after quantisation. In this way we see that the Myers interactions in the WZ term arise very naturally.

4. Kappa-symmetry

One approach to kappa-symmetry for single branes is to note that both the DBI and WZ terms can be obtained from a closed $(p+2)$-form $W := (e^{-\mathcal{F}}G)_{p+2}$, where G denotes the sum of the RR field strengths, on the super worldvolume M. It is obvious that $W = dL_{\rm WZ}$, where $L_{\rm WZ} = (e^{-\mathcal{F}}C)_{p+1}$ for a single brane, and it can be shown by cohomological methods that W is exact, in fact that $W = -dL_{\rm DBI}$. It therefore follows that

$$\mathcal{L} := L_{\rm DBI} + L_{\rm WZ} \tag{4.1}$$

is a closed $(p+1)$-form on M. One can therefore use "ectoplasmic" integration [22] to obtain an action which will be invariant under local (i.e. kappa) supersymmetry [23, 24]; this is given by

$$\int \varepsilon^{m_1\ldots m_{p+1}} \mathcal{L}_{m_1\ldots m_{p+1}}(x, \theta = 0)\,, \tag{4.2}$$

where the integral is taken over M_0, the bosonic worldvolume of the brane. If we now make a supersymmetry transformation on M, i.e. an odd diffeomorphism with parameter κ^α, we find

$$\delta\mathcal{L} = i_\kappa d\mathcal{L} + d(i_\kappa\mathcal{L}) = d(i_\kappa\mathcal{L})\ . \tag{4.3}$$

Evaluating (4.3) at $\theta = 0$ and applying it in the variation of (4.2) we get the desired result. Kappa-symmetry is essentially local supersymmetry on the super worldvolume; the usual kappa parameter is defined by

$$\kappa^{\underline{\alpha}} = \kappa^\alpha E_\alpha{}^{\underline{\alpha}} \tag{4.4}$$

evaluated at $\theta = 0$.

This construction can be extended to the non-abelian case in a more or less straightforward manner. We shall show directly that

$$-dL_{\rm DBI} \simeq W = e^{-K}G \tag{4.5}$$

where the modified equals sign indicates equality up to terms that vanish in the Bernstein-Leites integral. Since the generalisation of the "ectoplasm" construction is straightforward, establishing (4.5) will be sufficient to prove kappa-symmetry. Note that the kappa-symmetry parameter in this case will depend on ξ as well as x; in this sense we have non-abelian kappa-symmetry as well. In fact, we need only consider terms with at least one factor of E^α since such a factor is needed to contract with κ.

We begin by evaluating $dL_{\rm DBI}$. We have

$$\begin{aligned} d\varepsilon_{(p+1)} &= \frac{1}{p!}E^{a_p}\ldots E^{a_1}T^c\varepsilon_{ca_1\ldots a_p} \\ &\simeq \frac{1}{p!}E^{a_p}\ldots E^{a_1}\left(\frac{1}{2}E^\beta E^\alpha T_{\alpha\beta}{}^c + E^b E^\alpha T_{\alpha b}{}^c\right)\varepsilon_{ca_1\ldots a_p}\,, \end{aligned} \tag{4.6}$$

where in this equation, and for the rest of this section, the $\simeq$ sign indicates equality up to terms that either integrate to zero or which do not have at least one factor of E^α. Making use of (2.34) and (2.36) we obtain

$$d\varepsilon_{(p+1)} \simeq -\frac{i}{2}\varepsilon_a E^\beta E^\alpha[(\gamma^a(h^{-1})^T + h\gamma^a)h^T]_{\alpha\beta} + i\varepsilon_{(p+1)}E^\alpha(h\gamma^a h_a)_\alpha\,, \tag{4.7}$$

where

$$\varepsilon_a := \frac{1}{p!}E^{b_p}\ldots E^{b_1}\varepsilon_{ab_1\ldots b_p} \tag{4.8}$$

Let us now consider the derivative of the $e^{-\mathcal{I}}$ factor. It is easy to see that

$$\begin{aligned} d\mathcal{I} &= E^{\dot\gamma}T_{\dot\gamma} \\ &\simeq \frac{1}{2}E^{\dot\gamma}E^{\beta}E^{\alpha}T_{\alpha\beta\dot\gamma} + E^{\dot\gamma}E^{\dot\beta}E^{\alpha}T_{\alpha\dot\beta\dot\gamma}\,, \end{aligned} \tag{4.9}$$

where the other terms in $T^{\dot\gamma}$ have been dropped because they will not contribute to the integral of $i_\kappa dL_{\mathrm{DBI}}$. (We remind the reader that dotted indices are raised or lowered using $\delta^{\dot\alpha\dot\beta}$ or $\delta_{\dot\alpha\dot\beta}$.) Using the expressions for the torsion is (2.35) and (2.37) we find

$$d\mathcal{I} \simeq \frac{i}{2}E^{\dot\gamma}E^{\beta}E^{\alpha}[(\gamma_{\dot\gamma}(h^{-1})^T - h\gamma_{\dot\gamma})h^T]_{\alpha\beta} - iE^{\dot\gamma}E^{\dot\beta}E^{\alpha}(h\gamma_{\dot\gamma}h_{\dot\beta})_\alpha\ . \tag{4.10}$$

When we integrate over $E^{\dot\alpha}$ the second term will give rise to a contraction between the $\dot\beta$ and $\dot\gamma$ indices in the last factor, so that we can replace (4.10) by

$$d\mathcal{I} \simeq \frac{i}{2}E^{\dot\gamma}E^{\beta}E^{\alpha}[(\gamma_{\dot\gamma}(h^{-1})^T - h\gamma_{\dot\gamma})h^T]_{\alpha\beta} - iE^{\alpha}(h\gamma^{\dot\beta}h_{\dot\beta})_\alpha\ . \tag{4.11}$$

We also need to evaluate the derivative of L_0. We have

$$dL_0 \simeq \frac{1}{2}L_0E^{\alpha}\left(((\eta+\mathcal{F})^{-1})^{cb}\nabla_\alpha\mathcal{F}_{bc} - ((1+\eta)^{-1})^{\dot\gamma\dot\beta}\nabla_\alpha\eta_{\dot\beta\dot\gamma}\right)\ . \tag{4.12}$$

With the aid of (2.38) and (2.39) we obtain

$$((\eta+\mathcal{F})^{-1})^{cb}\nabla_\alpha\mathcal{F}_{bc} = i\left(-(h\gamma^a h_a)_\alpha + (h\gamma_a h_b)_\alpha L^{ba}\right)\,, \tag{4.13}$$

where L_{ab} is given in (2.28), as well as

$$((1+\eta)^{-1})^{\dot\gamma\dot\beta}\nabla_\alpha\eta_{\dot\beta\dot\gamma} = i\left((h\gamma^{\dot\beta}h_{\dot\beta})_\alpha - (h\gamma_{\dot\beta}h_{\dot\gamma}L^{\dot\gamma\dot\beta})_\alpha\right)\,, \tag{4.14}$$

where

$$L_{\dot\alpha}{}^{\dot\beta} := (1+\eta)_{\dot\alpha}{}^{\dot\gamma}((1-\eta)^{-1})_{\dot\gamma}{}^{\dot\beta}\ . \tag{4.15}$$

This L is an element of SO(q), where q is the number of fermions. Since

$$h\gamma_a h^T = \gamma^b L_{ba} \tag{4.16}$$

we have

$$h\gamma_a L^{ba} = \gamma^b(h^{-1})^T\ . \tag{4.17}$$

We can also show that

$$h\gamma_{\dot\alpha}L^{\dot\beta\dot\alpha} = -\gamma^{\dot\beta}(h^{-1})^T \tag{4.18}$$

This can be seen as follows: we have

$$\begin{aligned} h\gamma_{\dot\alpha}h^T &= h_{\dot\alpha}{}^{a'}h\gamma_{a'}h^T \\ &= -h_{\dot\alpha}{}^{a'}\gamma^{b'}L_{b'a'} \\ &= -h_{\dot\alpha}{}^{a'}\gamma^{b'}[(1+M)(1-M)^{-1}]_{b'a'}\ . \end{aligned} \tag{4.19}$$

On the other hand

$$\begin{aligned} h_{\dot\alpha}{}^{a'} M_{a'b'} &= h_{\dot\alpha}{}^{a'} h_{\dot\beta a'} \delta^{\dot\beta\dot\gamma} h_{\dot\gamma b'} \\ &= \eta_{\dot\alpha}{}^{\dot\beta} h_{\dot\beta b'} \ . \end{aligned} \tag{4.20}$$

It is then a short step to verify (4.18).

Combining all the above results and taking into account the dilaton factor in $L_{\rm DBI}$ we finally arrive at

$$\begin{aligned} dL_{\rm DBI} \simeq \ & \frac{1}{2} L_{\rm DBI} E^\alpha \big(-2\nabla_\alpha\phi + i[(h\gamma^a h_a)_\alpha + \gamma^a (h^{-1})^T h_a)_\alpha] \\ & \qquad\qquad +i[(h\gamma^{\dot\alpha} h_{\dot\alpha})_\alpha - (\gamma^{\dot\alpha}(h^{-1})^T h_{\dot\alpha})_\alpha]\big) \\ & -\frac{i}{2} e^{-\phi} e^{-\mathcal{I}} L_0 E^\beta E^\alpha \big(\varepsilon_a [(\gamma^a (h^{-1})^T + h\gamma^a) h^T]_{\alpha\beta} \\ & \qquad\qquad +\varepsilon_{(p+1)} E^{\dot\alpha} [(\gamma_{\dot\alpha}(h^{-1})^T - h\gamma_{\dot\alpha}) h^T]_{\alpha\beta}\big) \ . \end{aligned} \tag{4.21}$$

We now turn to the Wess-Zumino form. We begin by proving that

$$e^{-\phi} e^{-K} \sum \gamma^{(2n)} \simeq -L_{\rm DBI} h \tag{4.22}$$

Consider the terms in the l.h.s. of (4.22) which involve $\mathcal{F}^m$ and which have $(p+1)$ factors of E^a. If we set $n = k + l$, where $2m + 2k = p + 1$, then we get terms of the form

$$\left(e^{-\phi} \frac{(-1)^m}{2^m m!} E^{a_{2m}} \dots E^{a_1} \mathcal{F}_{a_1 \dots a_{2m}} \right) \left(\frac{1}{(2k)!} E^{b_{2k}} \dots E^{b_1} \gamma_{b_1 \dots b_{2k}} \right) \left(\frac{1}{(2l)!} E^{\dot\alpha_{2l}} \dots E^{\dot\alpha_1} \gamma_{\dot\alpha_1 \dots \dot\alpha_{2l}} \right) , \tag{4.23}$$

where

$$\mathcal{F}_{a_1 \dots a_{2m}} := \mathcal{F}_{[a_1 a_2} \dots \mathcal{F}_{a_{2m-1} a_{2m}]} \ . \tag{4.24}$$

Using

$$E^{a_{p+1}} \dots E^{a_1} = -\varepsilon^{a_1 \dots a_{p+1}} \varepsilon_{(p+1)} \tag{4.25}$$

and the duality relation

$$\gamma^{a_1 \dots a_{2m}} \gamma_{(p+1)} = \frac{(-1)^m}{(p+1-2m)!} \varepsilon^{a_1 \dots a_{p+1}} \gamma_{a_{2m+1} \dots a_{p+1}} \tag{4.26}$$

we find that the first two factors in (4.23) give

$$-\varepsilon_{(p+1)} \frac{1}{2^m m!} \gamma^{a_1 \dots a_{2m}} \mathcal{F}_{a_1 \dots a_{2m}} \gamma_{(p+1)} \ . \tag{4.27}$$

When we integrate the third factor in (4.23) over $E^{\dot\alpha}$, taking into account the presence of $e^{-\mathcal{I}}$ in W, we find

$$\begin{aligned} \int D(E^{\dot\alpha})\, e^{-\mathcal{I}} \frac{1}{(2l)!} E^{\dot\alpha_{2l}} \dots E^{\dot\alpha_1} \gamma_{\dot\alpha_1 \dots \dot\alpha_{2l}} &= \frac{1}{2^l l!} \delta^{\dot\alpha_1 \dot\alpha_2} \dots \delta^{\dot\alpha_{2l-1} \dot\alpha_{2l}} \gamma_{\dot\alpha_1 \dots \dot\alpha_{2l}} \\ &= \frac{1}{2^l l!} \gamma^{a'_1 \dots a'_{2l}} M_{a'_1 \dots a'_{2l}} \ , \end{aligned} \tag{4.28}$$

where

$$M_{a'_1\ldots a'_{2l}} := M_{[a'_1a'_2}\ldots M_{a'_{2l-1}a'_{2l}]}\ . \tag{4.29}$$

Putting all this together, summing over all terms of the type of (4.23) and recalling the series expression for h we indeed find (4.22). When the first index on $\gamma^{(2n)}$ is a superscript, a similar calculation yields

$$e^{-\phi}e^{-K}\sum\widetilde{\gamma}^{(2n)} \simeq L_{\rm DBI}(h^{-1})^T\ . \tag{4.30}$$

We can now show that the terms involving $\nabla\phi$ in W sum up to give the corresponding term in $-dL_{\rm DBI}$. The relevant term in W is

$$\begin{aligned}-e^{-\phi}e^{-K}&\left(E^{\alpha1}(\gamma^{(2n)}\nabla_2\phi)_\alpha-(-1)^nE^{\alpha2}(\gamma^{(2n)}\nabla_1\phi)_\alpha\right)\simeq\\ &\simeq -e^{-\phi}e^{-K}E^\alpha\left((\gamma^{(2n)}\nabla_2\phi)_\alpha-(-1)^n(h\gamma^{(2n)}\nabla_1\phi)_\alpha\right)\ .\end{aligned} \tag{4.31}$$

Using the facts that

$$(-1)^n(\gamma^{(2n)})_\alpha{}^\beta = (\gamma^{(2n)})^\beta{}_\alpha\,, \tag{4.32}$$

and $\nabla_\alpha = E_\alpha{}^{\underline{\alpha}}\nabla_{\underline{\alpha}}$ together with (4.22) and (4.30) we indeed see that this term gives $L_{\rm DBI}E^\alpha\nabla_\alpha\phi$ as required.

The remaining terms in W we need to consider, when pulled back to $\widehat{M}$, have the form

$$ie^{-\phi}e^{-K}E^\alpha\left(E^\gamma h_\gamma{}^\beta+E^ch_c{}^\beta+E^{\dot\gamma}h_{\dot\gamma}{}^\beta\right)(\gamma^{(2n-1)})_{\alpha\beta}\ . \tag{4.33}$$

The easiest term to deal with is the one involving $h_a{}^\beta$. We have

$$E^a\gamma^{(2n-1)} = -\frac{1}{2}[\gamma^a,\gamma^{(2n)}]\ . \tag{4.34}$$

Using this, (4.22) and (4.30), we easily find that these terms give

$$-\frac{i}{2}L_{\rm DBI}((h\gamma^ah_a)_\alpha+(\gamma^a(h^{-1})^Th_a)_\alpha)\,, \tag{4.35}$$

which is what we needed to show. Now consider the term involving $h_{\dot\gamma}{}^\beta$. We shall compute this directly. The terms that involve $\mathcal{F}^m$ will require $2k$ factors of E^a from $\gamma^{(2n-1)}$, where $2m+2k=p+1$, as well as an odd number, say $2l+1$, of $E^{\dot\alpha}$ terms. The E^a contribution is the same as (4.27). The $E^{\dot\alpha}$ contribution comes from terms of the form

$$\frac{1}{(2l+1)!}E^{\dot\alpha_{2l+2}}\ldots E^{\dot\alpha_1}\gamma_{\dot\alpha_1\ldots\dot\alpha_{2l+1}}h_{\dot\alpha_{2l+2}}{}^\beta\ . \tag{4.36}$$

After integration this gives

$$\frac{1}{2^ll!}\delta^{\dot\alpha_1\ldots\dot\alpha_{2l}}\gamma_{\dot\alpha_1\ldots\dot\alpha_{2l}\dot\gamma}\delta^{\dot\gamma\dot\delta}h_{\dot\delta}{}^\beta\ . \tag{4.37}$$

Writing $\gamma_{\dot\alpha_1\ldots\dot\alpha_{2l}}{}^{\dot\delta}=\frac{1}{2}\{\gamma_{\dot\alpha_1\ldots\dot\alpha_{2l}},\gamma^{\dot\delta}\}$, using the multi-trace to convert the dotted indices to primed vector indices, and summing all such contributions we find

$$-\frac{i}{2}L_{\rm DBI}E^\alpha\left((h\gamma^{\dot\alpha}h_{\dot\alpha})_\alpha-(\gamma^{\dot\alpha}(h^{-1})^Th_{\dot\alpha})_\alpha\right) \tag{4.38}$$

which matches minus the third term in the first line of (4.21). Finally, we need to examine the terms with $E^\alpha E^\beta$. Since E^β pulls back to both e^b and $e^{\dot\beta}$ there are two contributions; the former will require an odd number of factors of E^a to be selected from $\gamma^{(2n-1)}$ while the latter will require an even number. In both cases the calculations are very similar to the ones we have already done. The term with an odd number of E^as will give rise to a total of p of them when combined with the $\mathcal{F}$ terms and thus gives rise to a factor ε_a. It is not difficult to verify that it gives precisely minus the first term on the second line of (4.21). The other term is also easily calculated. It gives

$$\frac{i}{2}L_{\rm DBI}E^\alpha e_{\dot\alpha}{}^\beta\left((\gamma^{\dot\alpha}(h^{-1})^T - h\gamma^{\dot\alpha})h^T\right)_{\alpha\beta}\ . \tag{4.39}$$

This should match minus the second term on the second line of (4.21). This is

$$\frac{i}{2}L_{\rm DBI}E^\alpha E^\beta E^{\dot\alpha}\left((\gamma_{\dot\alpha}(h^{-1})^T - h\gamma_{\dot\alpha})h^T\right)_{\alpha\beta}\ . \tag{4.40}$$

In this expression we may replace E^β by $e^{\dot\gamma}e_{\dot\gamma}{}^\beta$, and then the integral over $D(e^{\dot\alpha})$ forces a contraction between the $\dot\gamma$ and $\dot\alpha$ indices. Thus we obtain (4.39).

This completes the proof that $i_\kappa W \simeq -i_\kappa dL_{\rm DBI}$ and shows that the action $\int(L_{\rm DBI} + L_{\rm WZ})$ is indeed kappa-symmetric.

5. Discussion

In this paper we have constructed an action for coincident D-branes using the boundary fermion formalism in the classical approximation. As we argued in our previous papers, naive quantisation of the fermions after going to the physical gauge leads to the Myers action (in the bosonic sector) with the integral over the fermions replaced by the symmetrised trace. Myers started from the non-abelian generalisation of Born-Infeld [25, 26] and deduced the form of the scalar terms, in the physical gauge, by demanding T-duality. He also used T-duality as a guiding principle for his construction of the WZ term. Similar results were independently derived from matrix model considerations [10, 11]. It is known, however, that this action and its supersymmetric generalisation proposed here, is not the full story; see, for example [27, 28]. There have been various attempts to derive these corrections systematically, including the stable bundle approach [29], direct attempts to construct non-commutative differential geometry [30 − 33] and others [34 − 39]. It would certainly be of interest to try to develop the boundary fermion formalism further to see if contact can be made with these ideas.

The main achievement of the current paper is the supersymmetrisation of our action for bosonic branes. This was made much easier by the use of Bernstein-Leites integration; the action given here also has the virtue of being manifestly covariant under all of the relevant symmetries, with the exception of kappa-symmetry. However, the proof of the latter, as we have seen, is very similar to the proof of kappa-symmetry for a single brane. It is interesting to note that the kappa-symmetry parameter depends on the boundary fermions and thus becomes matrix-valued when they are quantised. This is in accord with

JHEP09(2007)010

the ideas of references [40, 41]. Other attempts to supersymmetrise non-abelian brane dynamics have usually assumed that there is a single kappa-symmetry. These include supersymmetric Born-Infeld actions [42, 43], studies of higher-derivative component actions in ten-dimensional Yang-Mills theory [44], investigations of $N = 4, D = 4$ higher-order actions in superspace [45], $N = 4, D = 4$ terms from $N = 1$ supergraphs [46] and attempts to incorporate non-abelian terms in the superembedding formalism [47, 48]. There is a possible intermediate gauge choice we could make which would be to fix the non-abelian worldvolume coordinate and kappa-symmetries leaving one kappa-symmetry and one diffeomorphism intact; this could then lead to comparisons with the one-kappa approaches to the problem we have just mentioned.

Acknowledgments

This paper was supported in part by EU-grant (Superstring Theory) MRTN-2004-512194. UL acknowledges partial support from VR grant 621-2006-3365.

References

[1] P.S. Howe, U. Lindström and L. Wulff, *Superstrings with boundary fermions*, *JHEP* **08** (2005) 041 [hep-th/0505067].

[2] P.S. Howe, U. Lindström and L. Wulff, *On the covariance of the Dirac-Born-Infeld-Myers action*, *JHEP* **02** (2007) 070 [hep-th/0607156].

[3] N. Marcus and A. Sagnotti, *Group theory from quarks at the ends of strings*, *Phys. Lett.* **B 188** (1987) 58.

[4] H. Dorn and H.J. Otto, *On t-duality for open strings in general abelian and nonabelian gauge field backgrounds*, *Phys. Lett.* **B 381** (1996) 81 [hep-th/9603186].

[5] P. Kraus and F. Larsen, *Boundary string field theory of the DD-bar system*, *Phys. Rev.* **D 63** (2001) 106004 [hep-th/0012198].

[6] N. Berkovits and V. Pershin, *Supersymmetric Born-Infeld from the pure spinor formalism of the open superstring*, *JHEP* **01** (2003) 023 [hep-th/0205154].

[7] I.A. Bandos, D.P. Sorokin, M. Tonin, P. Pasti and D.V. Volkov, *Superstrings and supermembranes in the doubly supersymmetric geometrical approach*, *Nucl. Phys.* **B 446** (1995) 79 [hep-th/9501113].

[8] P.S. Howe and E. Sezgin, *Superbranes*, *Phys. Lett.* **B 390** (1997) 133 [hep-th/9607227].

[9] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053].

[10] I.W. Taylor and M. Van Raamsdonk, *Multiple D0-branes in weakly curved backgrounds*, *Nucl. Phys.* **B 558** (1999) 63 [hep-th/9904095].

[11] I.W. Taylor and M. Van Raamsdonk, *Multiple Dp-branes in weak background fields*, *Nucl. Phys.* **B 573** (2000) 703 [hep-th/9910052].

[12] J.N. Bernstein and D.A. Leites, *Integral forms and the Stokes formula on supermanifolds*, *Funktsional. Anal. I Prilozhen.* **11** (1977) 55 (Russian) [English translation: *Funct. Anal. Appl.* **11** (1977) 45]; *Integration of differential forms on supermanifolds*, *Funktsional. Anal. I Prilozhen.* **11** (1977) 71 [English translation: *Funct. Anal. Appl.* **11** (1977) 221].

[13] L. Wulff, *Strings, boundary fermions and coincident D-branes*, hep-th/0701129.

[14] C.S. Chu, P.S. Howe and E. Sezgin, *Strings and D-branes with boundaries*, *Phys. Lett.* **B 428** (1998) 59 [hep-th/9801202].

[15] P.S. Howe and P.C. West, *The complete $N = 2$, $D = 10$ supergravity*, *Nucl. Phys.* **B 238** (1984) 181.

[16] N. Berkovits and P.S. Howe, *Ten-dimensional supergravity constraints from the pure spinor formalism for the superstring*, *Nucl. Phys.* **B 635** (2002) 75 [hep-th/0112160].

[17] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super-p-branes in ten-dimensional type-IIA and IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159].

[18] I. Bandos and D. Sorokin, *Aspects of D-brane dynamics in supergravity backgrounds with fluxes, kappa-symmetry and equations of motion. IIB*, *Nucl. Phys.* **B 759** (2006) 399 [hep-th/0607163].

[19] C.G. Callan Jr., C. Lovelace, C.R. Nappi and S.A. Yost, *Loop corrections to superstring equations of motion*, *Nucl. Phys.* **B 308** (1988) 221.

[20] A. Belopolsky, *De rham cohomology of the supermanifolds and superstring BRST cohomology*, *Phys. Lett.* **B 403** (1997) 47 [hep-th/9609220].

[21] P.A. Grassi and G. Policastro, *Super-Chern-Simons theory as superstring theory*, hep-th/0412272.

[22] S.J. Gates Jr., M.T. Grisaru, M.E. Knutt-Wehlau and W. Siegel, *Component actions from curved superspace: normal coordinates and ectoplasm*, *Phys. Lett.* **B 421** (1998) 203 [hep-th/9711151].

[23] I.A. Bandos, D.P. Sorokin and D. Volkov, *On the generalized action principle for superstrings and supermembranes*, *Phys. Lett.* **B 352** (1995) 269 [hep-th/9502141].

[24] P.S. Howe, O. Raetzel and E. Sezgin, *On brane actions and superembeddings*, *JHEP* **08** (1998) 011 [hep-th/9804051].

[25] A.A. Tseytlin, *On non-abelian generalisation of the Born-Infeld action in string theory*, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[26] A. Hashimoto and I.W. Taylor, *Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action*, *Nucl. Phys.* **B 503** (1997) 193 [hep-th/9703217].

[27] Y. Kitazawa, *Effective lagrangian for open superstring from five point function*, *Nucl. Phys.* **B 289** (1987) 599.

[28] A. Bilal, *Higher-derivative corrections to the non-abelian Born-Infeld action*, *Nucl. Phys.* **B 618** (2001) 21 [hep-th/0106062].

[29] P. Koerber and A. Sevrin, *The non-abelian D-brane effective action through order α'^4*, *JHEP* **10** (2002) 046 [hep-th/0208044].

[30] J. De Boer and K. Schalm, *General covariance of the non-abelian DBI-action*, *JHEP* **02** (2003) 041 [hep-th/0108161].

[31] L. Cornalba, M.S. Costa and R. Schiappa, *D-brane dynamics in constant Ramond-Ramond potentials and noncommutative geometry*, *Adv. Theor. Math. Phys.* **9** (2005) 355 [hep-th/0209164].

[32] D. Brecher, K. Furuuchi, H. Ling and M. Van Raamsdonk, *Generally covariant actions for multiple D-branes*, *JHEP* **06** (2004) 020 [hep-th/0403289].

[33] D. Brecher, P. Koerber, H. Ling and M. Van Raamsdonk, *Poincaré invariance in multiple D-brane actions*, *JHEP* **01** (2006) 151 [hep-th/0509026].

[34] N. Wyllard, *Derivative corrections to D-brane actions with constant background fields*, *Nucl. Phys.* **B 598** (2001) 247 [hep-th/0008125].

[35] S.F. Hassan and R. Minasian, *D-brane couplings, RR fields and Clifford multiplication*, hep-th/0008149.

[36] N. Wyllard, *Derivative corrections to the D-brane Born-Infeld action: non-geodesic embeddings and the Seiberg-Witten map*, *JHEP* **08** (2001) 027 [hep-th/0107185].

[37] L. Anguelova and P.A. Grassi, *Super D-branes from BRST symmetry*, *JHEP* **11** (2003) 010 [hep-th/0307260].

[38] S.F. Hassan, $N = 1$ *worldsheet boundary couplings and covariance of non-abelian worldvolume theory*, hep-th/0308201.

[39] R. Schiappa and N. Wyllard, *D-brane boundary states in the pure spinor superstring*, *JHEP* **07** (2005) 070 [hep-th/0503123].

[40] E.A. Bergshoeff, M. de Roo and A. Sevrin, *Non-abelian Born-Infeld and kappa-symmetry*, *J. Math. Phys.* **42** (2001) 2872 [hep-th/0011018].

[41] E.A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, *Supersymmetric non-abelian Born-Infeld revisited*, *JHEP* **07** (2001) 029 [hep-th/0105274].

[42] S. Gonorazky, F.A. Schaposnik and G.A. Silva, *Supersymmetric non-abelian Born-Infeld theory*, *Phys. Lett.* **B 449** (1999) 187 [hep-th/9812094].

[43] S.V. Ketov, $N = 1$ *and* $N = 2$ *supersymmetric nonabelian Born-Infeld actions from superspace*, *Phys. Lett.* **B 491** (2000) 207 [hep-th/0005265].

[44] A. Collinucci, M. De Roo and M.G.C. Eenink, *Supersymmetric Yang-Mills theory at order* α'^3, *JHEP* **06** (2002) 024 [hep-th/0205150].

[45] J.M. Drummond, P.J. Heslop, P.S. Howe and S.F. Kerstan, *Integral invariants in* $N = 4$ *SYM and the effective action for coincident D-branes*, *JHEP* **08** (2003) 016 [hep-th/0305202].

[46] A. Refolli, A. Santambrogio, N. Terzi and D. Zanon, F^5 *contributions to the nonabelian Born-Infeld action from a supersymmetric Yang-Mills five-point function*, *Nucl. Phys.* **B 613** (2001) 64 [hep-th/0105277].

[47] D.P. Sorokin, *Coincident (super)-Dp-branes of codimension one*, *JHEP* **08** (2001) 022 [hep-th/0106212].

[48] J.M. Drummond, P.S. Howe and U. Lindström, *Kappa-symmetric non-abelian Born-Infeld actions in three dimensions*, *Class. and Quant. Grav.* **19** (2002) 6477 [hep-th/0206148].

JHEP09(2007)010